STAT 2593

Lecture 023 - Some General Concepts of Point Estimation

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Some General Concepts of Point Estimation

Learning Objectives

1. Understand and differentiate between estimators and estimates.
2. Understand methods for evaluating point estimates, including the MSE, bias, and variance.
3. Describe the MVUE.
4. Define the standard error.

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  - We refer to this **single value** as a **point estimate**, or **estimate** more broadly.
- The **function** that takes in the sample and produces the *estimate* is called an **estimator**.

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  - The estimate will depend on both the estimator used, and on the sample.

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  - We can use the **mean squared error**, or MSE, given by $\text{MSE} = E\left[(\widehat{\theta} - \theta)^2\right]$.
  - The MSE has a **bias-variance** decomposition, where $\text{MSE} = \text{Bias}(\widehat{\theta})^2 + \text{var}(\widehat{\theta})$.

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  - Note that, unbiasedness is generally not carried through functions (e.g., sample standard deviation).
  - Some estimators are unbiased as $n \to \infty$. These are said to be **asymptotically unbiased**.

A Side Note on the Sample Variance

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- **Going forward, we will use this altered form!**

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- If we consider *only* unbiased estimators, minimizing variance can provide us with the **best** unbiased estimator.
  - We refer to this as the **minimum variance unbiased estimator**, or MVUE.

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  - The sample mean will *not* generally be the MVUE for the population mean.
  - We say that an estimator is **robust** if it performs well for many distributions.

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- Often the standard error will rely on unknown parameters, and so we will use an **estimated standard error**.
  - Sometimes we cannot find analytical expressions, where other strategies (such as the bootstrap) need to be employed.
- The estimated standard error should **always** be reported, alongside a point estimate, to estimate the spread.

Summary

- Estimates are single values which represent a guess as to a parameter value in a population.
- Estimators are random variables which produce estimates.
- Estimators are assessed through their MSE, bias, and variance.
- Unbiasedness is preferable, generally, but not always.
- The MVUE is the best unbiased estimator.
- Standard errors quantify uncertainty in our point estimates, and should be reported.